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ABSTRACT

An exact multivariate analysis for troublesome repeated measures designs has been described by Bock and programmed by Finn. The method is applied to digit span from an actual experiment involving first-grade pupils in an inner-city school and a suburban school in Canada. The repeated measures are first transformed by an orthogonal matrix derived from the design on the measures; the resulting new variables are treated as dependent variables in the multivariate analysis of variance employing the design on the subjects. In this example, Bock's method yielded more significant results compared to conventional approximate analyses. Covariates may be used. (Author)

Multivariate Analysis of Repeated Measures with a Design on the Measures and a Design on the Subjects--An Example[1]

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Repeating observations on the same subject has always been a boon to experimenters and a headache to statisticians. Sometimes measures repeated over time are the essence of the study, as in learning and developmental studies, but just as often experimenters feel that several observations of the same person are meaningful or preferable, as in using the subject as his own control. Pretest-posttest seems the rule rather than the exception. Whether two or twenty, repeated observations lack the characteristic, statistical independence, crucial to every inferential procedure save those which concentrate on the correlations themselves.

Neophytes and Fisherphobes who run the non-parametric domain find that independence rules that kingdom no less sternly, perhaps more so for lack of any rivals. And these soldiers of the king prove not so strong.

We grant the strength and clear result where independence prevails, especially where backed by randomization. Every raw recruit in the statistician army can be a general with that weapon in his arsenal. But too few experimenters are willing to pay the price to achieve randomization, that is, careful advance planning and extensive talking to persuade people that the value is worth the effort. In the face of repeated measures, how can we best manoeuvre when complete victory is beyond our grasp? Do we attack, dig in--or retreat?

In this paper we describe a double frontal attack on some particular repeated measures data, an attack which can be mounted whenever the measures have a factorial structure, i.e., wherever they have been

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gathered according to some factorial design. The method is applicable, however, wherever by design or by theory it is plausible to argue that the data have some structure expressible in well behaved mathematical form.

The concept is by no means new, but practical execution requires sophisticated computer programs and we lack experience in the interpretation of the results. A sophisticated computer program is at hand in Multivariance (version 4-1, Finn, 1971) and a full elaboration of the method will soon be published (Bock in press). We present an application of the method that reveals some of its power and some of its challenges.

A Simple Example

Because the example to be discussed is somewhat messy, we begin with a simple example which illustrates the essence of the attack, namely transformation of the repeated (non-independent) measures into new meaningful, orthogonal variables before attempting statistical inference.

Suppose one tests a group of boys and a group of girls on two geometry problems. The solutions to the geometry problems yield two repeated measures on the two independent groups. There is a design on the measures (simple one-way design, problems A and B) and also on the subjects (also one-way, boys vs girls).

Sufficient statistics for the usual normal-theory analysis are the four means for problems and sexes plus the variances and covariances (or sums of squares and cross products, of course). Look at the four means first:

$$
\begin{array}{c c}
 & \text{Means} \\
 & \begin{array}{cc} \text{Problem A} & \text{Problem B} \end{array} \\
\begin{array}{c} \text{Boys} \\ \text{Girls} \end{array} & \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & \bar{Y}_{22} \end{bmatrix}
\end{array}
$$

Though there are four cells (4n observations if n boys and n girls) there are not 4n independent degrees of freedom.

The analysis is simplified if the matrix of means is transformed by a simple 2 x 2 matrix T,

$$T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

which converts the two repeated measures into two other "measures", namely their sum and difference. We now have

	Sum	Difference
Boys	$Y_{11}+Y_{12}$	$Y_{11}-Y_{12}$
Girls	$Y_{21}+Y_{22}$	$Y_{21}-Y_{22}$

It is easy to verify that whatever the correlation between A and B, the sum and difference are uncorrelated. Furthermore, these two new dependent variables (sum and difference) are themselves meaningful. Separate univariate analysis of variance would be informative, but a pooled analysis is more powerful,.as shown in table 1.

Table 1

Interpretation of the Univariate Analysis of Variance of Sum and Difference Scores

Source	df	meaning of F-test
Mean	1	Geometry Problems
Sum		Tests whether grand mean is zero. Not interesting.
Difference		Geometry problem main effect: mean of differences
Groups (boys-girls)	1	Sex x Problem Interactions
Sum		Tests sex main effect
Difference		Sex by Problem interaction: difference of differences
Error	n_1+n_2-2	Pooled estimate of error

The essential feature of this technique is the use of an orthogonal transformation to convert correlated measures to more tractable ones before statistical analysis. With more than two measures the new transformed variables are not usually uncorrelated but the main procedures are identical. It is necessary to do a multivariate analysis of the transformed measures, taking account of the entire covariance structure, not just the variances. Where there is also a design on the subjects, the type of statistical tests one can make depends upon the extent of equivalence in the covariance structure, i.e., the multivariate analog of homogeneity of variance. This first illustration is too simple to reveal the essence of the method, but it does show how the main effects of the repeated measures appear in the first half of the table and the interactions with the design on the subjects appear in the second half. A "sex effect" in the difference scores is exactly what one means by a "sex-problem interaction".

The Main Example

The data that serve as the basis for our example were obtained in a study conducted by Keeton (1973). The design is described in Table 2. Exploratory data analysis led the author to create a new variable, Scoring, namely the number of digits recalled correctly from the first half of the series (primacy) and from the second half (recency). Results were averaged for the three trials at each of the lengths 4 and 5, 6 and 7, 8 and 9 to arrive finally at the data labelled 4, 6 and 8 in the sequel. (The middle digit was ignored for odd numbered lengths.) Full details are available in Keeton (1973).

These aggregated repeated measures are reported in tables 3 and 4 for the two independent groups of pupils, one from an inner-city (low SES) school and one from a suburban (high SES) school. Thus, tables 3 and 4 reveal the 2 x 2 x 3 design on the repeated measures and together illustrate the simple one-way design on the subjects.

The hypothesis of primary interest was that the inner-city children would score higher on the second half of the series than the suburban children and vice versa for the first half. In analysis of variance terminology, there would be a significant SES x Scoring interaction. A number of other hypotheses made a complex analysis desirable.

The Transformation and the Data

A 12 x 12 orthogonal matrix was generated that reflected the 2 x 2 x 3 design on the repeated measures. Multivariance (Finn, 1971) contains a feature that makes this a simple matter. The data are entered into

the program as 12 different dependent variables, taking care that the order of entry corresponds to the 2 x 2 x 3 design. In this example, there are exactly 50 children in each group, yielding two 12 x 1 vectors of means. Applying the 12 x 12 transformation matrix yields 12 new variables for each group corresponding to the main effects and interactions of a 2 x 2 x 3 analysis of variance. The entries in the actual matrix are fractional quantities dictated by the requirement of orthogonality, but the pattern is a familiar one; sample rows are shown to one decimal place in table 5. The structure of the vector of means is shown for comparison so that the meaning of the new "variables" can be seen. Because one of the factors, length, has ordered, equally-spaced levels, it suggests a model in which orthogonal polynomials are used to study the linear and quadratic components of the variability due to length. (In fact, such a model accounts for slightly more variance than one containing only simple difference contrasts.) Use of orthogonal polynomial coefficients accounts for the presence of zeros in table 5.

Results

A complete summary table is shown as table 6. Comparison with table 1 shows the same pattern of new variables within group effects, the lower half representing all the Group x (new variable) interactions. Both tests of mean vectors (Multivariate) show significance beyond .01, an almost guaranteed result for the constant term because of the grand mean itself. The significant test of groups as a whole shows that a fairly complex pattern exists in the data.

Looking at the top half (constant), we see all but two univariate tests significant beyond .01. This cannot be taken at face value because as we noted, when we go beyond two repeated measures the new variables are not uncorrelated. A look at the cross-product terms shows a number of large entries relative to the sums of squares. We have to turn to the step-down tests, a series of step-wise regression results with all variables above a given one included in the regression equation. Note that the new variable scoring (P-R) is not even significant if we remove the effect of the grand mean! This odd result is marked in left and right margins with θ.

Only Mode and Scoring x Mode survive the step-down analysis in the top group. Looking below, however, we see several interactions with Mode, so we will not pursue the main effect. Before going farther, however, let us acknowledge that the step-down tests are order sensitive and ask the readers to accept the authors' word that various orders were tried and the same robust effect emerged each time. Compliments are due Multivariance (i.e. Finn) again, in that repeated analyses changing only the order of the dependent variables (or their number) are both easy and economical of computer time. This is possible because only the very last stage of the analysis need be redone, using most calculations over again.

The effect of principal interest, SES x Scoring, is the first one below the line in the bottom group, that is, G x (P-R). It is significant both in univariate and step-down tests, no matter which variables are entered before (above) it. The hypothesis is thus strongly and validly supported, untainted by approximate F-tests or other adhocery. An unusual reversal is marked by * , where G x (V-A) appears significant in the step-down but not in univariate. This is apparently an artifact, since it does not happen with other orders.

As evidence of the truth of the maxim, "Be kind to your data and your data will be kind to you," we offer the seemingly unlikely three way interactions (marked by *), G x (V-A) x L2 and G x S x L1. Looking at the sums of squares it is clear that the pooled L1 and L2 effects would be significant, so we plot the means in figure 1.

The interesting G x S effect already mentioned turns out to be only part of the story. There is a difference in the slopes of the lines (G x S x L1), and a very interesting difference too! The lo SES (inner-city) children show a remarkably different pattern in recency from their primacy scores as length of series increases, while the Hi SES (suburban) group is consistent. That finding led to a second study, being reported tomorrow afternoon (Keeton and McLean, 1973).

Mode of presentation (visual vs auditory) is confusing theoretically, confounded experimentally and hopeless statistically. The confounding occurred because the children spontaneously rehearsed the digits verbally in the auditory presentation but not in the visual and because auditory always followed visual. None of this explains why the suburban group did so poorly at the auditory task for series of length 8 and 9 (see figure 1, G x (V-A) x L). The graphs are plotted primarily to show that when a result appears artifactual in the analysis, e.g. G x (V-A), its graph shows nothing, and when a result appears robust, e.g. G x (V-A) x L, its graph reveals a likely source of effect. If we can't interpret the results, is that Darrell Bock's fault?

Conclusions

Given the availability of a powerful tool such as Multivariance, it seems doubtful we should ever analyze repeated measures other than as multivariate data. Among the extensions not mentioned is the possibility of using covariates, not usually possible because we have only one covariate measure per person and (by definition) several repeated measures. The covariates can be applied to each of the new "variables", though this takes some extra work. (The transformatiom matrix has to be entered via cards as part of a super-matrix and a p x p identity matrix, where p is the number of covariates.)

The careful student of Bock-Finn will notice that this method is applicable to a single group as well as to data from more complex designs. The authors now have data from a second year of the study reported here, promising a future paper on non-orthogonal multivariate analysis of covariance of repeated measures after orthogonal transformation marinated and served with sour cream.

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TABLE 2

Design of Study No. 1

		Mode of Presentation									
		Auditory					Visual				
Length of Series		2	3	4	...	9	2	3	4	...	9
School Type (SES)	Inner City (Low)					N = 50					
	Suburb (High)					N = 50					

Table 3

Means, Standard Deviations and Correlations

for Inner City (Low SES) Group

Raw Data (before transformation)

Scoring	Mode	Length	Mean	S.D.	First Half						Second Half					
					Visual			Auditory			Visual			Auditory		
					4	6	8	4	6	8	4	6	8	4	6	8
First Half	Visual	4-5	.48	.24												
		6-7	.20	.14	.24											
		8-9	.18	.18	.18	.14										
	Auditory	4-5	.39	.26	.21	-.02	.06									
		6-7	.18	.26	.28	.12	.10	-.12								
		8-9	.27	.31	.06	-.10	-.18	.04	.25							
Second Half	Visual	4-5	.62	.22	.37	.22	.02	.02	.08	.06						
		6-7	.73	.26	-.03	-.16	.06	-.00	-.30	-.18	.25					
		8-9	.75	.33	.02	.05	-.21	.02	-.30	-.17	.29	.42				
	Auditory	4-5	.46	.25	-.03	-.03	-.02	.32	.14	-.14	.16	.27	.24			
		6-7	.63	.39	.24	.23	-.01	.06	-.22	-.01	.53	.22	.35	.31		
		8-9	.66	.40	.12	.06	-.03	.03	.06	.00	.47	.29	.31	.17	.50	

Table 4

Means, Standard Deviations and Correlations

for Suburban (High SES) Group

Raw Data (before transformation)

					First Half						Second Half					
					Visual			Auditory			Visual			Auditory		
Scoring	Mode	Length	Mean	S.D.	4	6	8	4	6	8	4	6	8	4	6	8
First Half	Visual	4-5	.60	.25												
		6-7	.28	.25	.41											
		8-9	.34	.27	.37	.28										
	Auditory	4-5	.45	.24	.39	.37	.40									
		6-7	.40	.33	.21	.30	.23	.33								
		8-9	.37	.31	.21	.13	.25	-.05	.28							
Second Half	Visual	4-5	.68	.21	.33	.14	.03	.19	.25	.07						
		6-7	.66	.35	-.00	-.29	-.12	.20	-.18	-.38	.09					
		8-9	.67	.35	-.22	-.23	-.39	.04	-.18	-.23	.14	.32				
	Auditory	4-5	.55	.28	.30	.28	.13	.42	.44	-.23	.15	.22	.24			
		6-7	.56	.33	-.34	-.13	-.28	.07	-.30	-.30	-.13	.37	.43	.17		
		8-9	.40	.30	.31	.15	.02	.28	-.03	-.10	.12	.29	.17	.34	.17	

Table 5

Sample Rows of Orthogonal Transformation (one decimal place only) and Vector of Means Showing the Design on the Measures from which the Transformation was Generated

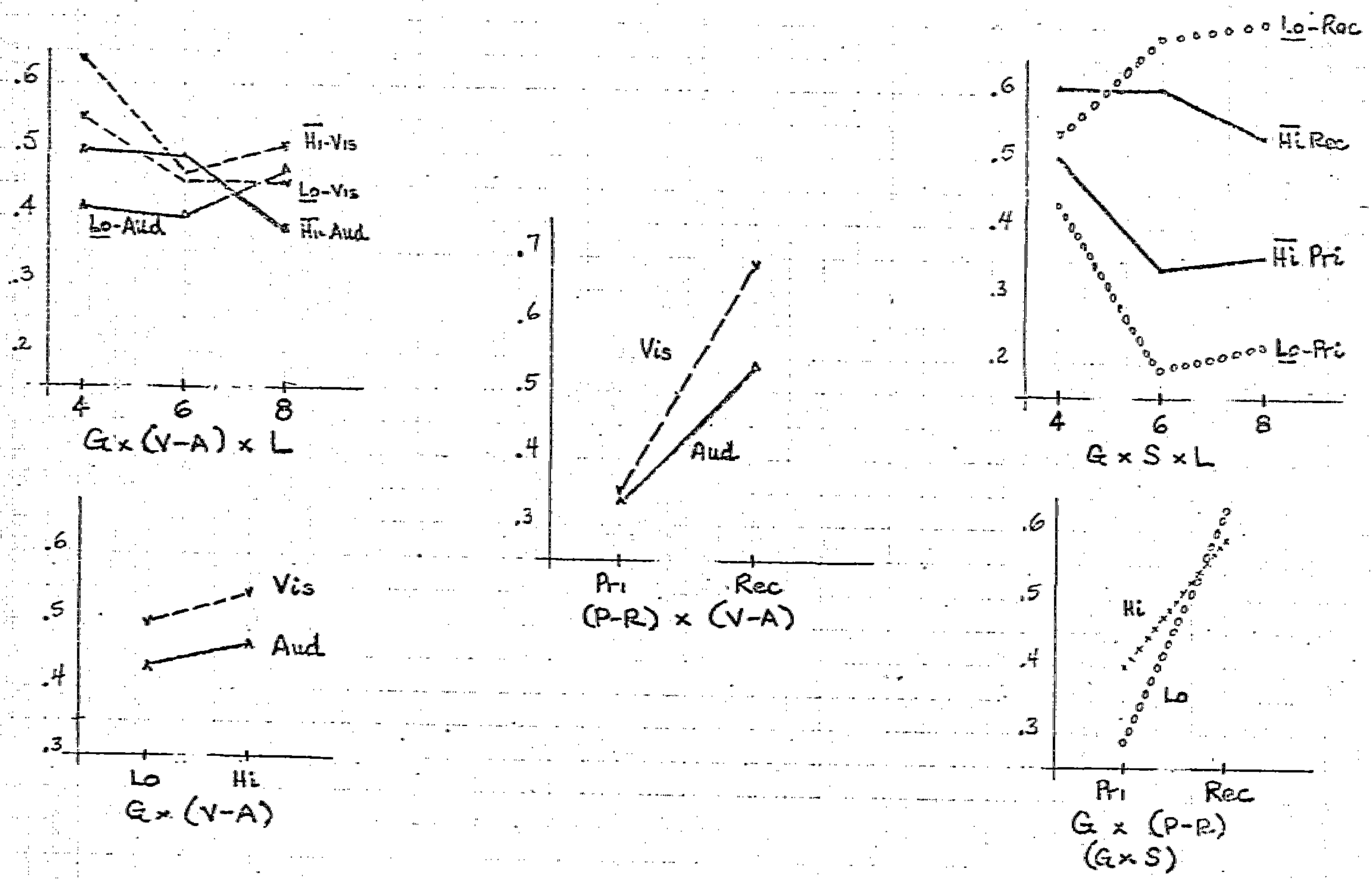
New Variables	Transformation Matrix												Suburban Mean Vector	Design: Length	Mode	Scoring
Mean	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.60	4	Vis	First Half (Primacy)
Scoring	.5	.5	.5	.5	.5	.5	-.5	-.5	-.5	-.5	-.5	-.5	.28	6		
Mode	.5	.5	.5	-.5	-.5	-.5	.5	.5	.5	-.5	-.5	-.5	.34	8		
Length 1	-.7	.0	.7	-.7	.0	.7	-.7	.0	.7	-.7	.0	.7	.45	4	Aud	
Length 2	.4	-.8	.4	.4	-.8	.4	.4	-.8	.4	.4	-.8	.4	.40	6		
M x L1													.37	8		
M x L2													.68	4	Vis	Second Half (Recency)
S x M	.2	.2	.2	-.2	-.2	-.2	-.2	-.2	-.2	.2	.2	.2	.66	6		
S x L1													.67	8		
S x L2													.55	4	Aud	
M x S x L1													.56	6		
M x S x L2	.1	-.2	.1	-.1	.2	-.1	-.1	.2	-.1	.1	-.2	.1	.40	8		

Values of new variables for Suburban group

Mean	Scoring	Mode	L1	L2	M x L1	M x L2	S x M	S x L1	S x L2	M x S x L1	M x S x L2
5.92	-.55	.27	-.36	.09	-.01	.12	-.12	-.08	.10	-.05	.02

Figure 1

Graphs of Mean Values
Corresponding to Significant Interaction Effects



Tabl: 6

Orthogonal Multivariate Analysis of Variance of
Transformed Digit Recall Scores

Design on the Measures : 2 x 2 x 3 . Scoring x Mode x Length

Design on the Subjects : one-way . Inner-city vs. Suburban Groups

Source of Dispersion	df	Sums of Squares and Cross-products												Multivariate Fo	Multivariate p	Univariate Fo	Univariate p	Step-Down Fo	Step-Down p
Constant (across groups)	1																		
Grand Mean		3288.07																	
@ Scoring: (P-R)		-463.75	65.41													114.52	.0001	0.00	.960 @
* Mode: (V-A)		128.78	-18.16	5.04												26.31	.0001	12.68	.00 *
Length: linear L1		-119.28	16.82	-4.67	4.33											18.06	.0001	.31	.58
: quad. L2		64.18	- 9.05	2.51	-2.33	1.25										6.95	.0098	.19	.66
(V-A) x L1		- 28.21	3.98	-1.11	1.02	- .55	.24									2.95	.0893	.03	.86
(V-A) x L2		35.02	- 4.94	1.37	-1.27	.68	-.30	.37								8.77	.0039	.14	.71
* (P-R) x (V-A)		- 59.13	8.34	-2.32	2.15	-1.15	.50	- .03	1.05					154.19	.0001	20.93	.0001	5.43	.02 *
(P-R) x L1		- 97.48	13.75	-3.82	3.54	-1.90	.84	-1.04	1.75	2.89						44.08	.0001	2.33	.13
(P-R) x L2		75.79	-10.69	2.97	-2.75	1.48	-.65	.81	-1.36	-2.25	1.75					23.94	.0001	.84	.36
S x M x L1		- 18.84	2.66	- .74	.68	- .37	.16	- .20	.34	.56	- .43	.11				8.25	.0050	.46	.49
S x M x L2		3.07	- .43	.12	- .11	.06	-.03	.03	- .06	- .09	.07	-.02	.00			.16	.6930	.28	.60
Groups	1																		
Between Groups		3.58														1.78	.1859	1.76	.19
* G x (P-R)		4.96	6.87													12.03	.0008	15.36	.00 *
⊙ G x (V-A)		.89	1.23	.22												1.16	.2849	4.02	.05 ⊙
G x L1		- 2.86	- 3.97	- .71	2.29											9.57	.0026	2.44	.12
G x L2		- .43	- .60	- .11	.34	.05										.29	.5925	2.19	.14
G x (V-A) x L1		.67	.93	.17	- .54	- .08	.13									1.55	.2166	1.38	.24
* G x (V-A) x L2		1.04	1.44	.26	- .83	- .12	.20	.30								7.10	.0091	4.04	.05 *
G x S x M		- .38	- .53	- .10	.31	.05	-.07	-.11	.04					3.16	.0009	.80	.3722	.46	.50
* G x S x L1		1.78	2.46	.44	-1.42	- .21	.34	.52	- .19	.88						13.47	.0004	3.79	.05 *
G x S x L2		- .52	- .73	- .13	.42	.06	-.10	-.15	.06	-.26	.08					1.05	.3072	.01	.93
G x S x M x L1		- .26	- .37	- .06	.21	.03	-.05	-.08	.03	-.13	.04	.02				1.50	.2240	.30	.58
G x S x M x L2		.24	.34	.06	- .20	- .03	.04	.07	- .03	.12	- .04	-.02	.02			.95	.3324	.02	.88
Within Groups Mean Square	98	2.02	.57	.19	.23	.18	.08	.04	.05	.07	.07	.01	.02						